RESEARCH ARTICLE

On the new wave behavior of the Magneto-Electro-Elastic(MEE) circular rod longitudinal wave equation

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ARTICLE INFO	ABSTRACT
	The analytical solution of the longitudinal wave equation in the MEE circular rod is analyzed by the powerful sine-Gordon expansion method. Sine - Gordon expansion is based on the well-known wave transformation and sine - Gordon equation. In the longitudinal wave equation in mathematical physics, the transverse Poisson MEE circular rod is caused by the dispersion. Some solutions with complex, hyperbolic and trigonometric functions have been successfully implemented. Numerical simulations of all solutions are given by selecting the appropriate parameter values. The physical meaning of the analytical solution explaining some practical physical problems is given.

1. Introduction

Innovative analytical new solutions for non-linear evolution equations (NEEs) has very important role in area of non-linear physics. Non-linear evolution equations are often used to state complex models that appear in different areas of non-linear science, such as biological sciences, quantum mechanics, and plasma physics. Recently, different analytical techniques have been invested to search new types of solutions. NLEs such as the new general algebra method [1], the $\tan(\frac{F(\xi)}{2})$-expansion method [2], the extended tanh method [3], the jacobi elliptic function method [4], the homogeneous balance method [5], the generalized Kudryashov method [6], the generalized (G'/G) method [7], the extended homoclinic test function method [8], the improved Bernoulli sub-equation function method [9], the improved exp $(-\Phi(\xi))$-expansion function method [10] and so on. In general, many more analytical techniques have been designed and used in obtaining analytical solutions of different NLEs [11–22]. Authors of

[23–28] obtained new lump and interaction for some of models in which arise in applied sciences. Moreover, Manafian and co-authors [29, 30] used the analytical methods for getting to exact solutions.

The powerful sine-Gordon expansion method (SGEM) [31, 32] was used to find some new solution methods to the longitudinal wave equation of the magneto-electro-elastic (MEE) circular rod [33] in this study. The longitudinal wave equation of the MEE circular rod is developed by [33], the longitudinal wave equation is a dispersion equation caused by the transverse Poisson's effect in MEE circular rod, developed from [34];

$$u_{tt} - q^2 u_{xx} - \left(\frac{q}{2}u^2 + pu_{tt}\right)_{xx} = 0, \qquad (1)$$

where p is the dispersion parameter and q is the linear longitudinal wave velocity of the MEE circular rod which depend on material properties and rod geometry [34]. Different analytical methods have been put in place to find solutions

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to the longitudinal wave equation in magneto-electro-elastic MEE circular rod, like the improved (G'/G)-expansion method [35], the functional variable method [36], the ansatz method [37], etc.

2. The SGEM

The general cases of SGEM was given in this section,

Take into account the following sine-Gordon equation [38], [39]:

$$u_{xx} - u_{tt} = n^2 sin(u). \qquad (2)$$

where $u = u(x, t)$ and $n \in \mathbb{R} \setminus \{0\}$.

Using the wave transformation $u = u(x, t) = U(\beta)$, $\beta = \alpha(x - kt)$ on Eq. (2), following non-linear ordinary differential equation (NODE) was gotten as:

$$U'' = \frac{n^2}{\alpha^2(1 - k^2)} sin(U), \qquad (3)$$

as $U = U(\beta)$, the amplitude of the traveling wave is β and k is the speed of the traveling wave. To integrate the equation (3), we get the following equation:

$$\left[\left(\frac{U}{2}\right)'\right]^2 = \frac{n^2}{\alpha^2(1 - k^2)} sin^2\left(\frac{U}{2}\right) + Q, \qquad (4)$$

as the integral constant is Q .

Set $Q = 0$, $\phi(\beta) = \frac{U}{2}$ and $b^2 = \frac{n^2}{\alpha^2(1-k^2)}$ in Eq. (4), gives:

$$\phi' = bsin(\phi), \qquad (5)$$

inserting $b = 1$ into Eq. (5), produces:

$$\phi' = sin(\phi), \qquad (6)$$

simplifying Eq. (6), creates the following two important equations;

$$sin(\phi) = sin(\phi(\beta)) = \left.\frac{2de^\beta}{d^2 e^{2\beta} + 1}\right|_{d=1} = sech(\beta), \qquad (7)$$

$$cos(\phi) = cos(\phi(\beta)) = \left.\frac{d^2 e^{2\beta} - 1}{d^2 e^{2\beta} + 1}\right|_{d=1} = tanh(\beta), \qquad (8)$$

as the integral constant is d .

For the given non-linear partial differential equation Eq. (9);

$$P(u, uu_x, u^2 u_t, \ldots), \qquad (9)$$

its solution in the form as;

$$U(\beta) = \sum_{i=1}^m tanh^{i-1}(\beta)\left[B_i sech(\beta) + A_i tanh(\beta)\right] + A_0. \qquad (10)$$

Equation (10) may be given according to Eq. (7) and (8) as;

$$U(\phi) = \sum_{i=1}^m cos^{i-1}(\phi)\left[B_i sin(\phi) + A_i cos(\phi)\right] + A_0. \qquad (11)$$

m is determined by balancing the highest power non-linear term and the highest derivative in the transformed NODE. Taking each summation of the coefficients of $sin^i(w)cos^j(w)$, $0 \le i, j \le m$ to be zero, produces a set of equations. This set of equation is solved with the symbolic computational computational software, yields the values of the coefficients A_i, B_i, μ and c. Eventually, inserting the produced values of these coefficients into Eq. (10) accompanied by the value of m, gives the fresh travelling wave solutions to Eq. (9).

3. Applications

The SGEM is used in searching the fresh solutions to Eq. (1) in this section. Considering Eq. (1), the following NODE was gotten by using the wave transformation; $u = U(\beta)$, $\beta = \mu(-kt + x)$;

$$2pk^2\mu^2 U'' - 2(k^2 - c_0^2)U + c_0^2 U^2 = 0, \qquad (12)$$

p is non-zero constant and we get $m = 2$ by balancing U'' and U^2 in Eq. (12).

Using Eq. (11) together with the value $m = 2$, we get the following equation;

$$\begin{aligned} U(\phi) &= B_1 sin(\phi) + A_1 cos(\phi) + B_2 cos(\phi)sin(\phi) \\ &\quad + A_2 cos^2(\phi) + A_0, \end{aligned} \qquad (13)$$

differentiating Eq. (13) twice, we get:

$$U''(\phi) = B_1 cos^2(\phi)sin(\phi) - B_1 sin^3(\phi)$$
$$- 2A_1 sin^2(\phi)cos(\phi) + B_2 cos^3(\phi)sin(\phi)$$
$$- 5B_2 sin^3(\phi)cos(\phi) - 4A_2 cos^2(\phi)sin^2(\phi)$$
$$+ 2A_2 sin^4(\phi),$$

$$(14)$$

Setting Eq. (13) and (15) to Eq. (12), generating trigonometric equations. After replacing the trigonometric constants in the trigonometric equation, a set of algebraic equations is collected by setting each sum of the coefficients of the trigonometric functions of the same power to zero. The set of equations is solved with assistance of symbolic mathematical softwares; to get coefficient values for different cases. We insert coefficient values for each case into the Eq. (10) with a value of $m = 2$, this gives us a new solution Eq. (1).

Case-1:

$$A_0 = 4(1 + \frac{k^2}{q^2}), A_1 = 0, B_1 = 0, A_2 = -6(1 - \frac{k^2}{q^2}),$$

$$B_2 = -6i + \frac{6ik^2}{q^2}, p = \frac{1}{k^2\mu^2}(k^2 - q^2).$$

Case-2:

$$A_0 = 4 - \frac{4}{1 + p\mu^2}, A_1 = 0, B_1 = 0,$$

$$A_2 = -6 + \frac{6}{1 + p\mu^2}, B_2 = \frac{6p\mu^2(p\mu^2 - 1)}{p^2\mu^4 - 1}i,$$

$$q = -k\sqrt{1 + p\mu^2}.$$

Case-3:

$$A_0 = -6 + \frac{6k^2}{q^2}, A_1 = 0, B_1 = 0, A_2 = 6 - \frac{6k^2}{q^2},$$

$$B_2 = 6i(1 - \frac{k^2}{q^2}), \mu = -\frac{1}{k\sqrt{p}}\sqrt{(k^2 - q^2)}.$$

Case-4:

$$A_0 = 1 + \frac{k^2}{q^2}, A_1 = 0, B_1 = 0, A_2 = -3(1 - \frac{k^2}{q^2}),$$

$$B_2 = 0, p = \frac{k^2 - q^2}{4k^2\mu^2}.$$

Case-5:

$$A_0 = 1 - \frac{1}{4p\mu^2 + 1}, A_1 = 0, B_1 = 0,$$

$$A_2 = -3 + \frac{3}{4p\mu^2 + 1}, B_2 = 0, q = k\sqrt{4p\mu^2 + 1}.$$

Case-6:

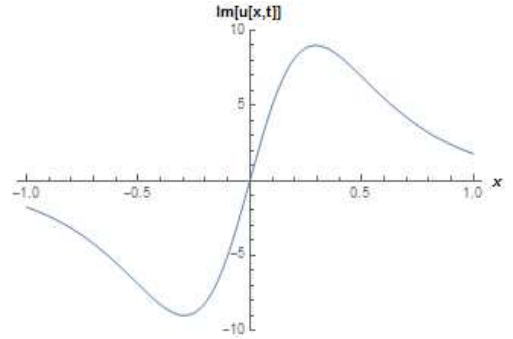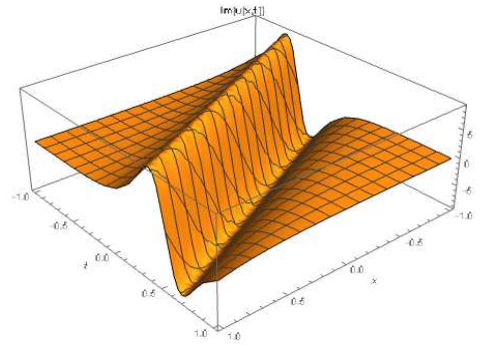$$A_0 = 1 - \frac{k^2}{q^2}, A_1 = 0, B_1 = 0, A_2 = 3(\frac{k^2}{q^2} - 1),$$

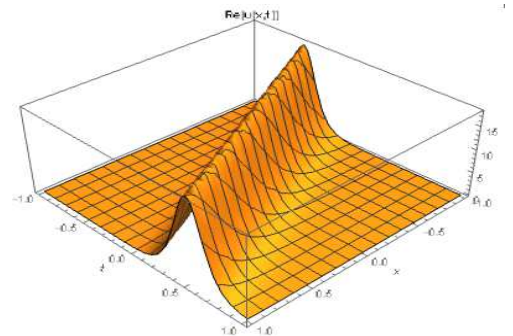$$B_2 = 0, \mu = \frac{1}{2k\sqrt{p}}\sqrt{(k^2 - q^2)}i.$$

Solutions:

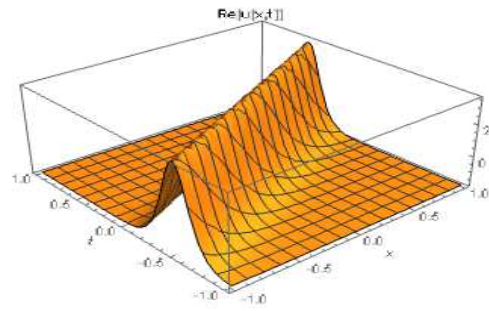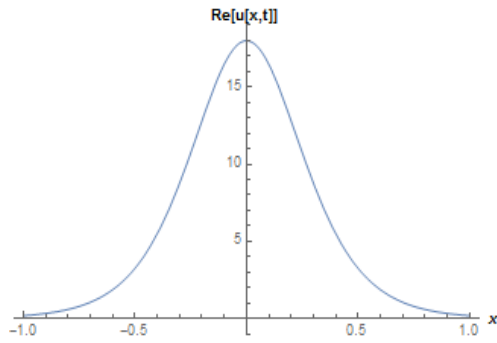**(1)**. The following solution is gotten by with case 1;

$$u_1(x, t) = \frac{6(k^2 - q^2)}{q^2}(1 + i\ sech[\mu(x - kt)]$$
$$\times tanh[\mu(x - kt)] - tanh[(-kt + x)\mu]^2)$$

$$(15)$$

Figure 1. The 3D shape for the imaginary part of Eq. (15) with the values $k = 2$, $c_0 = 1$, $\mu = 3$, $-3 < x < 3$, $-5 < t < 5$ and $t = 0$ for the graphic of 2D.
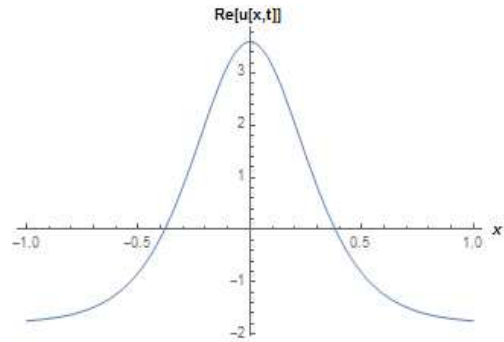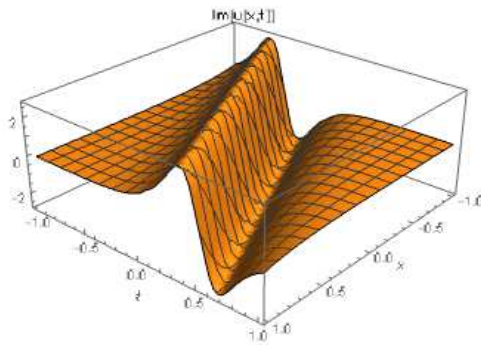
Figure 2. The 3D shape for the real part of Eq. (15) with the values $k = 2$, $c_0 = 1$, $\mu = 3$, $-3 < x < 3$, $-5 < t < 5$ and $t = 0$ for the graphic of 2D.

(2). The following solution is gotten by with case 2;

$$u_2(x,t) = 4 - \frac{4}{1 + p\mu^2} + (6ip\mu^2(-1 + p\mu^2)).$$
$$\cdot\frac{\sec h[(-kt + x)\mu]\tanh[-kt + x)\mu]}{-1 + p^2\mu^4}$$
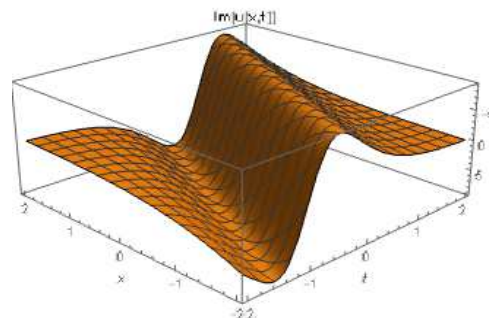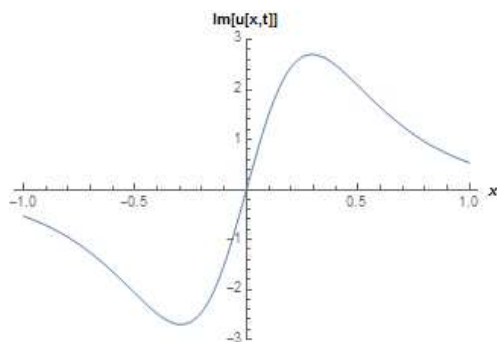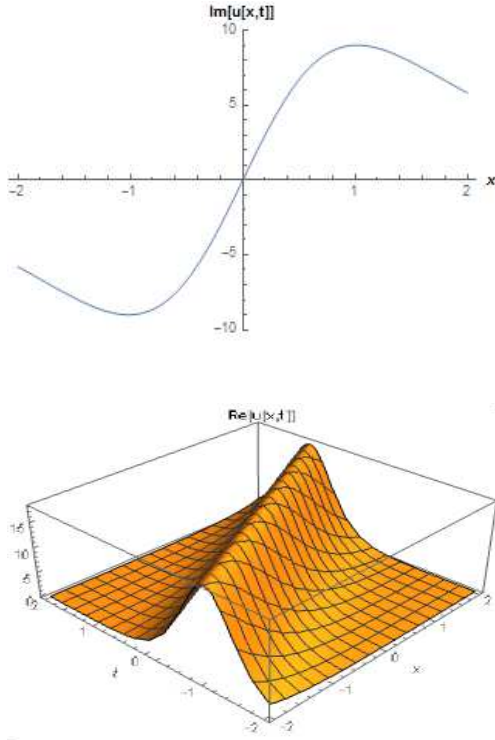$$+ (-6 + \frac{6}{1 + p^2\mu^2})\tanh[(-kt + x)\mu]^2.$$

$$(16)$$

Figure 3. The 2D and 3D shape for the imaginary and real part of Eq. (16) with the values $k = 2$, $p = 1$, $\mu = 3$, $-5 < x < 8$, $0 < t < 2$ and $t = 0$ for the graphics of 2D.

(3). The following solution is gotten by with case 3;

$$u_3(x,t) = \frac{6}{q^2}(q^2 - k^2)(-1 - i \ \sec h[\frac{1}{k\sqrt{p}}(\sqrt{k^2 - q^2})$$
$$\times (x - kt)]\tanh[\frac{1}{k\sqrt{p}}(\sqrt{k^2 - q^2})(x - kt)] +$$
$$+ \tanh[\frac{1}{k\sqrt{p}}(\sqrt{k^2 - q^2})(x - kt)]^2).$$

$$(17)$$

Figure 5. The 2D and 3D shape for the Eq. (18) with the values $k = 0.005$, $\mu = 3$, $c_0 = 1$, $-1 < x < 1$, $0 < t < 2$ and $t = 0$ for the graphic of 2D.

(5). The following solution is gotten by with case 5;

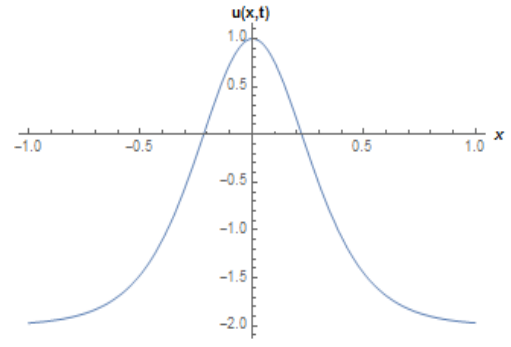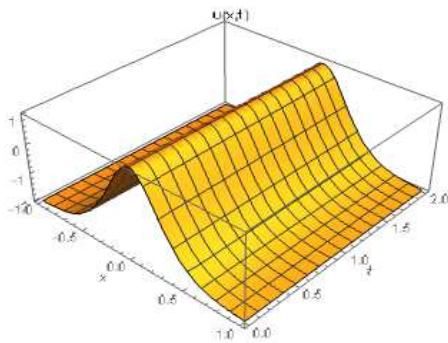$$u_5(x,t) = \frac{4p\mu^2}{1 + 4p\mu^2}(1 - 3\tanh[\mu(x - kt)]^2) \quad (19)$$

Figure 4. The 2D and 3D shape for the imaginary and real part of Eq. (17) with the values $k = 2$, $p = 1$, $c_0 = 1$, $-5 < x < 5$, $0 < t < 2$ and $t = 0$ for the graphics of 2D.

(4). The following solution is gotten by with case 4;

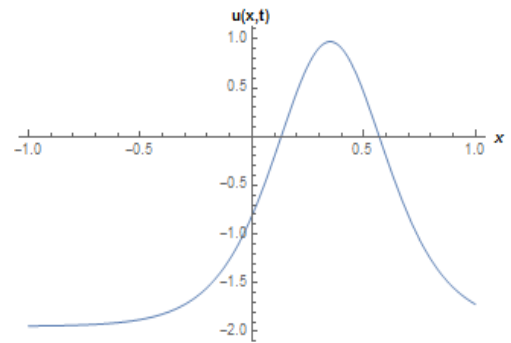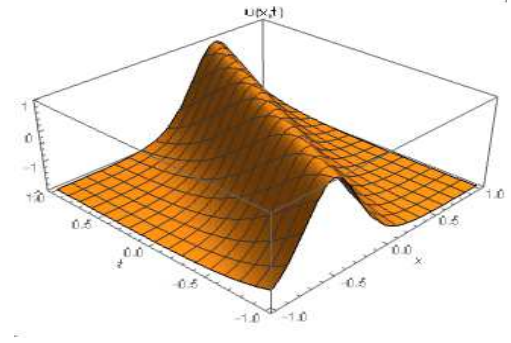$$u_4(x,t) = \frac{k^2 - q^2}{q^2}(2 - 3\tanh[(-kt + x)\mu]^2) \quad (18)$$

Figure 6. The 2D and 3D shape for the Eq. (19) with the values $k = 0.5$, $\mu = 3$, $p = 1$, $-0.5 < x < 1$, $0 < t < 2$ and $t = 0.7$ for the graphic of 2D.

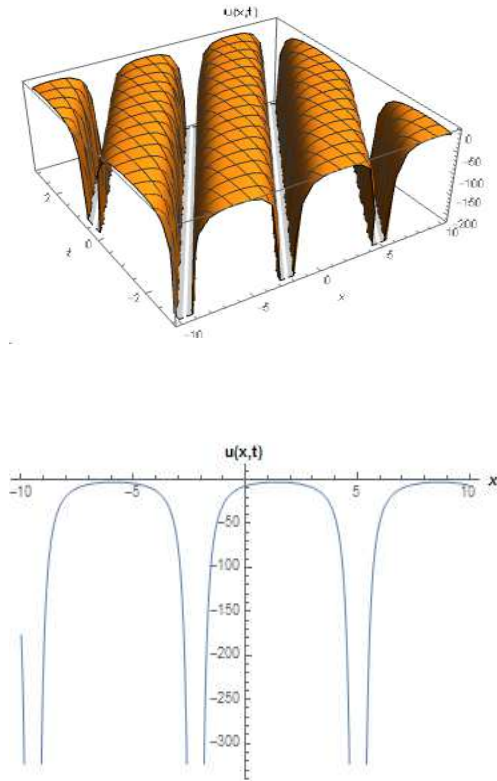**(6).** The following solution is gotten by with case 6;

$$u_6(x,t) = \frac{k^2 - q^2}{q^2}(-1 - 3\tan[\frac{\sqrt{k^2 - q^2}}{2k\sqrt{p}}(x - kt)]^2).$$

$$(20)$$

Figure 7. The 2D and 3D shape for the Eq. (20) with the values $k = 2$, $c_0 = 1$, $p = 1$, $-0.5 < x < 1$, $0 < t < 2$ and $t = 0.7$ for the graphic of 2D.

4. Results and Discussion

In [33] the improved $\exp(-\Phi(\xi))$-expansion function method is used in the solution of the magneto-electro-elastic circular rod longitudinal wave equation and the solution of different hyperbolic function forms is obtained. Secondly, the well-known improvement (G'/G)-expansion method [35] has been used for this equation and some precise hyperbolic and trigonometric functions are obtained. We observe that our results are new, but have the same solution structure. When compared with the existing, the results obtained by using these two methods. On the other hand, we observe that in the numerical simulations of the solutions we presented; Figure 1, Figure 2 and Figure 7 are singular soliton surfaces, Figure 3 is solit off surface, Figures 4-6 are soliton surfaces. We observe that some solutions in this study have important physical significance, such as the emergence of hyperbolic tangents in the calculation of magnetic moments and relative velocities, the emergence of hyperbolic secant in the profile of a laminar jet [40].

5. Conclusions

In this study, by utilizing the sine-Gordon extension method with the help of symbolic mathematical software, we investigated the solution of the magneto-electro-elastic circular rod longitudinal wave equation. We obtain some new solutions for complex hyperbolic and trigonometric functions. All solutions obtained in this study validate wave equations in magneto-electro-elastic circular rod and we examine this using the same procedure as symbolic mathematical software. We performed numerical simulations of all the solutions obtained in this paper. We observed that our results may be helpful in detecting transverse Poissons effect magneto-electro-elastic circular rod. The Sine-Gordon extension method is a powerful and efficient mathematical tool that can be used with the help of symbolic mathematical software to explore different non-linear methods arising in different fields of non-linear science.

Acknowledgments

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