RESEARCH ARTICLE

# The complex Ginzburg Landau equation in Kerr and parabolic law media

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| ARTICLE INFO | ABSTRACT |
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| | This paper study the complex Ginzburg-Landau equation with two different forms of nonlinearity. The Jacobi elliptic ansatz method is used to obtain the optical soliton solutions of this equation in the kerr and parabolic law media. Bright and dark optical soliton solutions are acquired as well as Jacobi elliptic function solutions. The existence criteria of these solutions are also indicated. |

## 1. Introduction

In recent years, studies conducted on findings new analytical solutions of differential equations have attracted attention of scientists from all over the world [1–21]. Especially the dynamics of optical soliton is one of the most fascinating areas of research in the field of mathematical physics. There are a great number of models that studies the dynamics of optical soliton propagation through a large variety of waveguides such as optical fibers, optical couplers, crystals, optical metamaterials and metasurfaces. The complex Ginzburg-Landau equation (CGLE) is one of these models and it is extended kind of the nonlinear Schrodinger equation that is the governing model of this context.The CGLE describes various phenomena including nonlinear optical waves, second-order phase transitions, Rayleigh–Bnard convection superconductivity, superfluidity, Bose–Einstein condensation and liquid crystals [1–4]. It is studied widely all over the world by a variety researchers [1–12]. A wealth of results have been reported in this context. Some

of the integration methods that have been implemented to this model are trial solution approach [7], modified simple equation method [8], first integral method [9], semi-inverse variational pirinciple [10] and others.The current paper will use Jacobi elliptic functions to extract cnoidal and snoidal wave solutions to the model.These will get soliton solutions in the limiting case of the modulus of ellipticity.

## 2. Mathematical analysis

The dimensions form of CGLE is [5]- [8]

$$iq_t + aq_{xx} + bF\left(|q|^2\right)q = \frac{1}{|q|^2 q^*}\left[\alpha |q|^2 \left(|q|^2\right)_{xx}\right.$$

$$\left. -\beta\left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q, \qquad (1)$$

where $q(x,t)$ is a complex-valued function which represents the soliton molecule in an optical fiber. The independent variables $x$ and $t$ show spatial and temporal coordinates, respectively. Then $a$ and $b$ represent coefficients of the group velocity dispersion (GVD) and nonlinearity, respectively.

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Also $\alpha$ and $\beta$ are additional nonlinear terms and $\gamma$ stem from the detuning effect [11].

In (1), if we think the complex plane $C$ as a two-dimensional linear space $R^2$, it can be written

$$F\left(|q|^2\right) q \in \cup_{\ell,n=1}^{\infty} C^k \left((-n,n) \times (-\ell,\ell)\, ; R^2\right). \tag{2}$$

The initial hypothesis for (1) is taken by the following form:

$$q(x,t) = u(\xi)\, e^{i\phi(x,t)}, \tag{3}$$

In (3), $u$ and $\phi$ represent amplitude and phase component of the soliton respectively and here

$$\xi = x - vt, \tag{4}$$

and

$$\phi = -\kappa x + wt + \theta \tag{5}$$

where $v$ represents the soliton velocity, $\kappa$ and $w$ represent the frequency and wave number of the soliton respectively and $\theta$ is the phase constant.

Substituting (3) into (1) and then decomposing real and imaginary parts, the real part is given

$$(a - 4\beta) u'' - \left(w + a\kappa^2 + \gamma\right) u + F\left(u^2\right) u$$
$$= 2\left(\alpha - 2\beta\right) \frac{(u')^2}{u}. \tag{6}$$

It is also note that $u' = du/d\xi$, $u'' = d^2u/d\xi^2$ and so on. The choice

$$\alpha = 2\beta, \tag{7}$$

Eq. (1) modifies to

$$iq_t + aq_{xx} + F\left(|q|^2\right) q = \frac{\beta}{|q|^2 q^*} \left[2|q|^2 \left(|q|^2\right)_{xx}\right.$$
$$\left. - \left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q, \tag{8}$$

and the real part reduces

$$(a - 2\alpha) u'' - \left(w + a\kappa^2 + \gamma\right) u + F\left(u^2\right) u = 0, \tag{9}$$

and then imaginary part of the Eq. (1) gives the soliton velocity as:

$$v = -2a\kappa. \tag{10}$$

The velocity of the soliton, given by (10), is independent of the type of nonlinearity. So it stays the same for all forms of fiber in question.

## 2.1. Kerr law

In this case,

$$F(s) = bs, \tag{11}$$

where $b$ is the real-valued constant. So, Eq. (8) reduces to

$$iq_t + aq_{xx} + \left(b|q|^2\right) q = \frac{\beta}{|q|^2 q^*} \left[2|q|^2 \left(|q|^2\right)_{xx}\right.$$

$$\left. - \left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q, \tag{12}$$

and the real part equation (9) simplifies to

$$(a - 4\beta) u'' - \left(w + a\kappa^2 + \gamma\right) u + u^3 = 0. \tag{13}$$

We assumed that $u$ is in the form

$$u(\xi) = A sn^\rho\left(B\xi, \ell\right), \quad \xi = x - vt, \tag{14}$$

where $\ell$ is the modulus of Jacobi elliptic function and $0 < \ell < 1$. Also $A$ represents the amplitude, $B$ is the inverse width of the soliton and unknown index $\rho$ will be determined.

Substituting Eq. (14) and its necessary derivatives in the real part Eq. (13), we have

$$(a - 4\beta)(\rho - 1)\rho AB^2 sn^{\rho-2}(B\xi, \ell)$$
$$- (a - 4\beta)\rho\left[\ell^2(\rho - 1) + \ell + \rho\right] AB^2 sn^\rho(B\xi, \ell)$$
$$+ (a - 4\beta)\ell\rho(\ell\rho + 1) AB^2 sn^{\rho+2}(B\xi, \ell) \tag{15}$$
$$\left(w + a\kappa^2 + \gamma\right) A sn^\rho(B\xi, \ell) + bA^3 sn^{3\rho}(B\xi, \ell) = 0.$$

From Eq.(15), matching the exponents $sn^{\rho+2}(B\xi, \ell)$ and $sn^{3\rho}(B\xi, \ell)$ yields

$$\rho + 2 = 3\rho, \tag{16}$$

which gives

$$\rho = 1. \tag{17}$$

Equating coefficients of them and setting coefficients of $sn^{\rho+j}(B\xi, \ell)$, for $j = -2, 0$, to zero in (15) as these are linearly independent functions yields

$$A = \sqrt{\frac{w + a\kappa^2 + \gamma}{b\ell}}, \tag{18}$$

$$B = \sqrt{\frac{w + a\kappa^2 + \gamma}{(4\beta - a)(\ell + 1)}}, \tag{19}$$

which requires the constraints

$$\left(w + a\kappa^2 + \gamma\right) b > 0, \tag{20}$$

$$\left(w + a\kappa^2 + \gamma\right)(4\beta - a) > 0. \tag{21}$$

So, for Kerr law nonlinearity, the Jacobi elliptic function solution is

$$q(x,t) = \sqrt{\frac{w + a\kappa^2 + \gamma}{b\ell}}.sn\left[\sqrt{\frac{w + a\kappa^2 + \gamma}{(4\beta - a)(\ell + 1)}}\right.$$
$$\left. (x + 2a\kappa t), \ell\right].e^{i(-\kappa x + wt + \theta)}, \tag{22}$$

If the modulus $\ell \to 1$ in Eq. (22), we obtain following dark optical soliton solution

$$q(x,t) = \sqrt{\frac{w + a\kappa^2 + \gamma}{b}}.\tanh\left[\sqrt{\frac{w + a\kappa^2 + \gamma}{2(4\beta - a)}}\right.$$
$$\left. (x + 2a\kappa t)\right].e^{i(-\kappa x + wt + \theta)}. \tag{23}$$

In solutions (22) and (23), $q(x,t)$ represents the soliton molecule in fiber. $\kappa$ and $w$ are the frequency and wave number of the soliton respectively, $\theta$ is the phase constant. Also $\gamma$ depicts detuning effect, $a$, $b$ and $\beta$ are constants.

In order to construct exact solutions for Eq. (12); we use hypothesis in the form

$$u(\xi) = A cn^{\rho}(B\xi, \ell),\qquad (24)$$

From (24), Eq. (13) reduces to

$$(a - 4\beta)(1 - \ell^2)(\rho - 1)\rho AB^2 cn^{\rho-2}(B\xi, \ell)$$

$$+ (a - 4\beta)\rho\left[\ell^2(2\rho - 1) + \ell - \rho\right]AB^2 cn^{\rho}(B\xi, \ell)$$

$$- (a - 4\beta)\ell\rho(\ell\rho + 1)AB^2 cn^{\rho+2}(B\xi, \ell)\quad (25)$$

$$- (w + a\kappa^2 + \gamma)Acn^{\rho}(B\xi, \ell) + bA^3 cn^{3\rho}(B\xi, \ell) = 0,$$

Setting the exponents and coefficients of functions $cn^{\rho+2}(B\xi, \ell)$ and $cn^{3\rho}(B\xi, \ell)$ equal to one another, and again setting the coefficients functions of $cn^{\rho+j}(B\xi, \ell)$ to zero for $j = -2, 0$, we acquire the same value of which is in (17) and following equations

$$A = \sqrt{\frac{\ell(\ell + 1)(w + a\kappa^2 + \gamma)}{b(\ell^2 + \ell - 1)}},\qquad (26)$$

$$B = \sqrt{\frac{w + a\kappa^2 + \gamma}{(a - 4\beta)(\ell^2 + \ell - 1)}},\qquad (27)$$

with the conditions

$$(w + a\kappa^2 + \gamma)b(\ell^2 + \ell - 1) > 0,\qquad (28)$$

$$(w + a\kappa^2 + \gamma)(a - 4\beta)(\ell^2 + \ell - 1) > 0.\quad (29)$$

Hence, we get the Jacobi elliptic function solution for CGLE with Kerr law nonlinearity as

$$q(x,t) = \sqrt{\frac{\ell(\ell + 1)(w + a\kappa^2 + \gamma)}{b(\ell^2 + \ell - 1)}}$$

$$.cn\left[\sqrt{\frac{w + a\kappa^2 + \gamma}{(a - 4\beta)(\ell^2 + \ell - 1)}}(x + 2a\kappa t), \ell\right]\quad (30)$$

$$.e^{i(-\kappa x + wt + \theta)}.$$

When $\ell \to 1$, solution (30) reduces bright optical soliton solution which is given by

$$q(x,t) = \sqrt{\frac{2(w + a\kappa^2 + \gamma)}{b}}$$

$$.\sec h\left[\sqrt{\frac{w + a\kappa^2 + \gamma}{(4\beta - a)}}(x + 2a\kappa t)\right]\qquad (31)$$

$$.e^{i(-\kappa x + wt + \theta)},$$

where $\kappa$ represents the soliton frequency, while $w$ depicts the wave number of the soliton. $\theta$, $a$, $b$ and $\beta$ are constants and so $\gamma$ arise from the detuning effect.

## 2.2. Parabolic law

In this case,

$$F(s) = b_1 s + b_2 s^2,\qquad (32)$$

where $b_1$ and $b_2$ are constants. So, Eq. (8) reduces to

$$iq_t + aq_{xx} + \left(b_1|q|^2 + b_2|q|^4\right)q = \frac{\beta}{|q|^2 q^*}$$

$$.\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q,\qquad (33)$$

and the real part Eq. (9) simplifies to

$$(a - 4\beta)u'' - (w + a\kappa^2 + \gamma)u + b_1 u^3 + b_2 u^5 = 0.\qquad (34)$$

The initial hypothesis as given below

$$u(\xi) = A\left[D + sn(B\xi, \ell)\right]^{\rho},\qquad (35)$$

So we get

$$(a - 4\beta)(\rho - 1)\rho AB^2(1 - D^2)(1 - \ell^2 D^2)$$

$$.\left[D + sn(B\xi, \ell)\right]^{\rho-2} + (a - 4\beta)\rho\left\{2\rho(1 - \ell^2 D^2)\right.$$

$$\left. + \ell(1 - D^2) + \ell^2(3D^2 - 2) - 1\right\}AB^2 D$$

$$.\left[D + sn(B\xi, \ell)\right]^{\rho-1} + (a - 4\beta)\rho$$

$$\left\{\ell D^2(6\ell\rho - 4\ell D + \ell + 2) + \ell^2(1 - 2D - \rho)\right.$$

$$\left. - \ell - \rho\right\}AB^2\left[D + sn(B\xi, \ell)\right]^{\rho} + (a - 4\beta)\quad (36)$$

$$.\ell\rho(-4\ell\rho + 3\ell - 3)AB^2 D\left[D + sn(B\xi, \ell)\right]^{\rho+1}$$

$$+ (a - 4\beta)\ell\rho(\ell\rho + 1)AB^2\left[D + sn(B\xi, \ell)\right]^{\rho+2}$$

$$- (w + a\kappa^2 + \gamma)A\left[D + sn(B\xi, \ell)\right]^{\rho}$$

$$+ b_1 A^3\left[D + sn(B\xi, \ell)\right]^{3\rho} + b_2 A^5\left[D + sn(B\xi, \ell)\right]^{5\rho} = 0.$$

Setting the exponents and the coefficients $\left[D + sn(B\xi, \ell)\right]^{\rho+1}$ and $\left[D + sn(B\xi, \ell)\right]^{3\rho}$ and also $\left[D + sn(B\xi, \ell)\right]^{\rho+2}$ and $\left[D + sn(B\xi, \ell)\right]^{5\rho}$ equal to one another, again equating the coefficients of $\left[D + sn(B\xi, \ell)\right]^{\rho+j}$ to zero, for $j = -2, -1, 0$, in the Eq. (36), yields

$$\rho = \frac{1}{2},\qquad (37)$$

$$D = \pm 1,\qquad (38)$$

$$A = \sqrt{\frac{b_1(\ell + 2)}{2b_2(\ell - 3)D}},\qquad (39)$$

$$B = 2\sqrt{\frac{(w + a\kappa^2 + \gamma)}{(a - 4\beta)\left[4\ell(1 + 2\ell - 2\ell D)\right.} },\qquad (40)$$
$$\overline{\left. + \ell^2(1 - 4D) - 2\ell - 1\right]}$$

with conditions

$$b_1 b_2(\ell - 3)D > 0,\qquad (41)$$

and

$$(w + a\kappa^2 + \gamma)(a - 4\beta)\left[4\ell(1 + 2\ell - 2\ell D)\right.$$

$$\left. + \ell^2(1 - 4D) - 2\ell - 1\right] > 0.\qquad (42)$$

Thus, the Jacobi elliptic function solution for the CGLE with parabolic law nonlinearity is given by

$$q(x,t) = \sqrt{\frac{b_1\,(\ell+2)}{2b_2\,(\ell-3)\,D}}.$$

$$\left[ D + sn\left(2\sqrt{\frac{(w+a\kappa^2+\gamma)}{(a-4\beta)\,[4\ell\,(1+2\ell-2\ell D)} \atop {+\ell^2\,(1-4D)-2\ell-1]}}\right.\right.$$
$$(x+2a\kappa t),\ell)\right].e^{i(-\kappa x+wt+\theta)}. \tag{43}$$

When the modulus $\ell \to 1$ , we obtain following dark optical soliton solution

$$q(x,t) = \frac{b_1}{2}\sqrt{\frac{-3}{b_2 D}}.$$

$$\left[ D + \tanh\left(\sqrt{\frac{2\,(w+a\kappa^2+\gamma)}{(a-4\beta)\,(5-6D)}}\right.\right. \tag{44}$$
$$(x+2a\kappa t))\right].e^{i(-\kappa x+wt+\theta)}.$$

In solutions (43) and (44), $q\,(x,t)$ represents the soliton molecule in fiber. $\kappa$ and $w$ are the frequency and wave number of the soliton respectively, $\theta$ is the phase constant. Also $\gamma$ depicts detuning effect, $a$, $b_1$, $b_2$, $D$ are constants.

Now, if we take the starting assumption as

$$u\,(\xi) = A\,[D + cn\,(B\xi,\ell)]^\rho, \tag{45}$$

Eq. (34) changes to

$$(a-4\beta)\,(\rho-1)\,\rho AB^2\,(1-D^2)\,(\ell^2+1)$$
$$.\,[D + cn\,(B\xi,\ell)]^{\rho-2} + (a-4\beta)\,\rho$$
$$\left\{[\ell^2(4\rho-3)+\ell]\,(D^2-1)+2\rho-1\right\}$$
$$.AB^2 D\,[D+cn\,(B\xi,\ell)]^{\rho-1}+(a-4\beta)\,\rho\,\{(1-3D^2)$$
$$(2\ell^2\rho-\ell^2+\ell)-\rho\}\,AB^2\,[D+cn\,(B\xi,\ell)]^\rho \tag{46}$$
$$+(a-4\beta)\,\ell\rho\,(4\ell\rho-\ell+3)\,AB^2 D\,[D+cn\,(B\xi,\ell)]^{\rho+1}$$
$$-(a-4\beta)\,\ell\rho\,(\ell\rho+1)\,AB^2 D\,[D+cn\,(B\xi,\ell)]^{\rho+2}$$
$$-(w+a\kappa^2+\gamma)\,A\,[D+cn\,(B\xi,\ell)]^\rho+b_1 A^3$$
$$[D+cn\,(B\xi,\ell)]^{3\rho}+b_2 A^5\,[D+cn\,(B\xi,\ell)]^{5\rho}=0.$$

Doing similar operations, value of the parameters $\rho$ and $D$ obtained the same as Eq.s (37) and (38) respectively and yields

$$A = \sqrt{\frac{-b_1\,(\ell+2)}{2b_2\,(\ell+3)\,D}}, \tag{47}$$

$$B = 2\sqrt{\frac{w+a\kappa^2+\gamma}{(a-4\beta)\,[2\ell\,(1-3D^2)-1]}}, \tag{48}$$

where

$$b_1 b_2 D > 0, \tag{49}$$
$$(w+a\kappa^2+\gamma)\,(a-4\beta)\,[2\ell\,(1-3D^2)-1] > 0. \tag{50}$$

So, we obtain

$$q(x,t) = \sqrt{\frac{-b_1\,(\ell+2)}{2b_2\,(\ell+3)\,D}}$$

$$\left[ D + cn\left(2\sqrt{\frac{w+a\kappa^2+\gamma}{(a-4\beta)\,[2\ell\,(1-3D^2)-1]}}\right.\right. \tag{51}$$
$$(x+2a\kappa t),\ell)\right].e^{i(-\kappa x+wt+\theta)}.$$

If the modulus $\ell \to 1$, we get following bright optical soliton solution

$$q(x,t) = \sqrt{\frac{-3b_1}{8b_2 D}}.$$

$$\left[ D + \sec h\left(2\sqrt{\frac{w+a\kappa^2+\gamma}{(a-4\beta)\,[1-6D^2]}}\right.\right. \tag{52}$$
$$(x+2a\kappa t))\right].e^{i(-\kappa x+wt+\theta)}.$$

where $q(x,t)$ represents the soliton molecule in fiber. $\kappa$ represents the soliton frequency, while $w$ depicts the wave number of the soliton. $\theta$, $a$, $b$ and $\beta$ are constants and so $\gamma$ arise from the detuning effect.

## 3. Conclusion

This paper consider CGLE in kerr and parabolic law media. Jacobi elliptic functions are used for the integration scheme here. Bright and dark optical soliton solutions are obtained using two types Jacobi elliptic functions. The existence criteria of these solutions are also indicated. These solutions provide recognise physical phenomena described by the equation. Due to the fact that bright and dark optical soliton solutions always help to address the soliton dynamics in long distance telecommunication system, the results of the paper are useful in the fiber optics communication technology. It can be obtained different solutions of the CGLE using the other Jacobi elliptic functions. This technique is very useful and effective to get soliton solutions of nonlinear partial differential equations in mathematical physics.

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