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RESEARCH ARTICLE

A belief-degree based multi-objective transportation problem with multi-choice demand and supply

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ABSTRACT

This paper focusses on the development of a Multi-choice Multi-objective Transportation Problem (MCMOTP) in the uncertain environment. The parameters associated with the objective functions in MCMOTP are regarded as uncertain variables and the other parameters associated with supply capacity and demand requirements are considered under the multi-choice environment. In this paper, two ranking criteria have been utilized to convert the uncertain objectives into their crisp form. Using these two ranking criteria for the uncertain MCMOTP model, two deterministic models have been developed namely, Expected Value Model (EV Model) and Optimistic Value Model (OV Model). The multi-choice parameters in the constraints are converted to a single choice parameters with the help of binary variable approach. The EV and OV models are solved directly in the LINGO 18.0 software using minimizing distance method and fuzzy programming technique. At last, a numerical illustration is provided to demonstrate the application and algorithm of the models. The sensitivity of the objective functions in OV Model is also examined with respect to the confidence levels to investigate variation in the objective functions.

1. Introduction

In the business sector, transportation is one of the most significant concerns. The Transportation Problem (TP) consists of many warehouses (sources) and delivery locations (destinations). The basic objective of this problem is to find the quantity of items that should be supplied from each warehouse to each consumer while reducing the transportation cost. The concept of TP was first introduced by Hitchock [1] in 1941. In real life applications, the decision-makers wish to optimize multiple objectives simultaneously instead of a single objective. Many researchers have addressed the multi-objective environment in a wide range of applications because it is better adapted to real-world scenarios than a single-objective environment. Some of the authors such as [2–4] have considered multi-objective environment to deal with various real world problems. When the multi-objective environment is incorporated in the transportation theory, it results in Multi-objective Transportation Problem (MOTP) which is more applicable in the real world. These multiple objectives can be inherently conflicting in nature, so they cannot be optimised at the same time. For example, a transportation problem may require minimising overall transportation cost and transportation time while transporting the items. Zimmermann [5] introduced the Fuzzy Programing Technique (FPT) to solve multi-objective linear programming problems. FPT has wide number of applications in the field of optimization problems such as [6–8]. The MOTPs can involve different types of multiple objective functions like transportation cost, transportation time, damage cost, profit, CO_2

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emissions etc.

In traditional TP, the supply, demand and cost parameters for transportation problems were assumed to be precisely defined. However, this is not the case while handling real-world applications because not all of the parameters in the TP can be defined precisely. Consequently, a number of theories have been developed to represent imprecise TP parameters such as fuzzy set theory [9], probability theory [10], and interval theory [11]. Transportation problems are distinguished by the parameter space or variable space in which they are defined. For example, the TPs with variables/parameters considered as fuzzy numbers are known as fuzzy TPs and the TPs with parameters considered in the random space are known as the stochastic TPs. Likewise, the TP with variables as interval numbers are known as interval TPs. These theories are suitable when historical information is available for estimating imprecise values. These notions (historical data available) have been studied by several researchers in the transportation problem. Many researchers like Bhargava et al. [12], Giri et al. [13], Ali Ebrahimnejad [14] have considered the multi-objective TP under the fuzzy logic and obtained the compromise solution using the different solution approaches. Maity et al. [15] presented the study of TP with interval goal under multiple objective environment and obtained the solution using utility function approach. Roy et al. [16] analysed MOTP under fuzzy intutionistic environment in his paper. Gupta et al. [17] presented the multi-objective capacitated TP under both the certain and uncertain environments and obtained their solution using fuzzy goal programming approach. Singh et al. [18] studied the three dimensional MOTP under the stochastic environment and the solutions were obtained using FPT. Gupta et al. [19] presented a paper on extended capacitated MOTP with mixed constraints under the fuzzy environment.

Research works cited so far gives the application of theories which are suitable when the historical data is available for the situations. In 2007, Liu [20] introduced a new theory for handling the imprecise or uncertain data known as Uncertainty theory. This theory is best suited in the situations where we face problems in accessing the historical data and have no adequate samples available. When there is lack of adequate samples, this theory deals with the degree of belief for each event to happen which is estimated by some experts of the related domain area. This theory finds a wide number of applications in the transportation problems. In the past years, a TP with uncertain values for time taken during transportation was considered by Mou et al. [21]. In 2017, a very commonly used goal programming approach was considered by Chen et al. [22] for solving a bi-objective Solid Transportation Problem (STP) under uncertainty to deal with an additional constraint for mode of transportation along with the source and destination constraints. Liu et al. [23] considered a solid TP with multiple items and fixed-charges under the assumption of data with uncertain numbers. Dalman [24] tackled an uncertain multi-item solid TP and obtained its solution using minimizing distance and convex-combination method. Mahmoodirad et al. [25] modelled a TP with fractional objectives where the uncertain parameters are taken as uncertain variables. Chen et al. [26] and Dalman [27] proposed an entropy based STP and multi-item STP in the uncertain environment. Recently, Zhao and Pan [28] in 2019 generalized the existing uncertain transportation models and proposed a new uncertain transportation model with transfer costs in which all the variables along with transfer costs are supposed to be uncertain numbers.

In addition to the imprecise parameter values of the TPs, it is also possible that multiple number of choices for a parameter are provided by the decision maker. In this context, the study of transport problems leads to the emergence of a new direction known as the multi-choice problem of multi-objective transport. In 2007, Chang [29] primarily introduced the multi-choice programming model. In his paper, he introduced a new idea of programming the problem having multiple choices for a parameter from which one choice of a parameter is to be selected and stated it as multi-choice goal programming. Biswal and Acharya [30] has done illustrious amount of work in the field of multi-choice theory which is capable of accommodating upto sixteen multi-choice parameters. Acharya et al. [31] gave generalized transformation techniques for multi-choice linear programming problems. Acharya and Biswal [32] solved the MCMOTP using interpolating polynomials and solved it using the fuzzy technique. The multi-choice TP under the stochastic environment has been studied by various researchers such as [33–35] with cost coefficients as multi-choice type and supply and demand parameters following different probabilistic distributions. Maity and Roy [36] obtained the solution of TP having non-linear cost and multi-choice demand in the multi-objective environment. Gupta et al. [37] have also studied the multi-choice multi-objective capacitated TP with the uncertain demand and supply. Roy et al. [38] introduced the conic scalarization approach to solve the multi-objective TP with an interval goal under a multi-choice environment. Nasseri [39] solved MOTP with multi-choice parameters where the alternative choices of the parameters were taken as random variables. They converted the multi-choice parameters into a single choice using the interpolating polynomials and obtained the solution by utilizing fuzzy approach. Nayak et al. [40] studied the TP with transportation cost as multi-choices and other parameters like supply and demand as fuzzy trapezoidal numbers. They used binary variable approach to choose a single choice among the multiple choices of the parameters. Agarwal et al. [41] presented a methodology for finding the solution of multi-choice TP with multi-choices as random variables. Vijayalakshmi et al. [42] contributed in the study of eco-friendly MOTP under the fuzzy environment and obtained its solution using goal programming approach. Agarwal and Ganesh [43] focussed on obtaining the solution of stochastic TP involving multi-choice random parameter using Newton's divided difference interpolation.

As seen from the literature survey, it is noted that the major amount of research work with multi-choice programming in TPs is mainly focused under the uncertain environments like stochastic or fuzzy or interval-valued environment (environments which require the historical data). Our paper proposes a new model, an uncertain multi-choice programming model for multi-objective transportation problem known as Uncertain Multi-choice Multi-objective Transportation Problem (UMCMOTP). The UMCMOTP considered in this paper is an extension of the basic transportation problem and we all are very much familiar that transportation problems find a wide number of applications in the economic and industrial and business sector for reducing the transportation cost and time, maximizing the profit etc. Such extension of the transportation problem (i.e MCMOTP) finds applications in the real world when we have multiple objective functions, various sources and destinations. Additionally, the MCMOTP with uncertain variables is more applicable in the real world when the decision maker finds difficulty in providing the precise value of the parameters associated with the objective functions and constraints due to various reasons like lack of information, weather conditions, road conditions etc. The uncertain multi-choice multi-objective transportation problem assumes uncertain variables in the objective functions and multi-choice parameters in the constraints. To obtain the solution of the UMC-MOTP, we have developed two different models: an EV Model and OV Model, by using concepts of uncertainty theory and multi-choice programming techniques. Further, the deterministic conversion of the uncertain objective functions is done by utilizing the expected and optimistic value criteria given by Liu [44] and the multi-choices in the constraints are converted to single choice using the binary variable approach suggested by Acharya and Biswal [30]. The formulated multi-objective models are then converted to single-objective models using the fuzzy programming and minimizing distance technique. Lastly, the compromise solution of the single-objective models is obtained with the help of LINGO 18.0 software.

The main motivations of this paper can be listed as below:

1. None of the researchers have considered the complex environments like uncertain multi-choice environment for transportation problem with objective parameters as uncertain variables and constraint parameters as multi-choice variables.
2. The expected value model has been widely adopted to convert the uncertain model into its crisp model whereas the optimistic value model has not yet been considered to deal with uncertain problems.
3. To solve the MOTP under uncertain multi-choice environment, we are the first to consider the minimizing distance method and fuzzy programming technique solution approaches.
4. The sensitivity of the objective functions with respect to confidence levels in the optimistic value model of multi-objective transportation problem under uncertain multi-choice environment has not yet been done.

The structure of the paper proceeds in the following manner. Section 2 discusses some key concepts in uncertainty theory that are essential for understanding this paper and Section 3 states the mathematical description of MCMOTP. Section 4 introduces the uncertain model for the MCMOTP and its conversion procedure for obtaining the deterministic model is discussed. Section 5 gives the two solution methodologies used for solving the deterministic models of MCMOTP. Section 6 provides a numerical illustration to depict the application of the models along with the sensitivity of the objective functions involved in the OV Model. Section 7 shows the obtained results and their comparison with other models. Finally, the last section gives the concluding remarks and summarizes the overall study of the paper.

2. Introduction to uncertainty theory

This section introduces some prime definitions and theoretical notions of uncertainty theory useful for the better understanding of the paper.

Definition 1. *(Liu [20]) A function $\mathcal{M}$: $\mathcal{F} \to [0,1]$ (here $\mathcal{F}$ is a σ-algebra defined on Ω and $\Omega \neq \phi$), is known as an uncertain measure if it meets the stated axioms:*

Axiom 1: $\mathcal{M}\{\Omega\} = 1$.
Axiom 2: $\mathcal{M}\{\upsilon\} + \mathcal{M}\{\upsilon^c\} = 1$, *for event* $\upsilon \in \mathcal{F}$.
Axiom 3: $\mathcal{M}\left\{\bigcup_{j=1}^{\infty} \upsilon_j\right\} \leq \sum_{j=1}^{\infty} \mathcal{M}\{\upsilon_j\}$, *for any countable sequence of events* $\{\upsilon_j\}$.

Here, the space denoted by the triplet $(\Omega, \mathcal{F}, \mathcal{M})$ is known as an uncertainty space.

Definition 2. *(Liu [20]) A measurable function ζ from $(\Omega, \mathcal{F}, \mathcal{M})$ to the real line $\mathcal{R}$ is said to be an uncertain variable if $\{\zeta \in \mathcal{B}\Big\}$ is an event for any Borel set $\mathcal{B}$ of real numbers.*

Definition 3. *(Liu [20]) The uncertainty distribution $\Psi : \mathcal{R} \to [0,1]$ for any uncertain variable ζ is defined as $\Psi(y) = \mathcal{M}\{\zeta \leq y\}$, for any real number y.*

Definition 4. *(Liu [20]) An uncertain variable ζ with $\Psi(y)$ defined as*

$$\Psi(y) = \begin{cases} 0, & if\, y \leq l, \\ \frac{y-l}{2(m-l)}, & if\, l \leq y \leq m, \\ \frac{y+n-2m}{2(n-m)}, & if\, m \leq y \leq n, \\ 1, & if\, y \geq n, \end{cases}$$

is called zigzag uncertain variable. Such uncertain variable ζ is characterised by $\mathcal{Z}(l,m,n)$ where l,m,n are any real numbers with $l < m < n$.

Definition 5. *(Liu [20]) The inverse uncertainty distribution function, denoted by Ψ^{-1} of $\mathcal{Z}(l,m,n)$ is given by*

$$\Psi^{-1}(\gamma) = \begin{cases} (1-2\gamma)l + 2\gamma m, & if\, \gamma < 0.5, \\ (2-2\gamma)m + (2\gamma - 1)n, & if\, \gamma \geq 0.5. \end{cases}$$

Theorem 1. *(Liu [20]) The expected value of an uncertain variable ζ, if it exists, is provided by*

$$E[\zeta] = \int_0^1 \Psi^{-1}(\gamma) d\gamma.$$

For the zigzag uncertain variable $\mathcal{Z}(l,m,n)$, the expected value is obtained as $E[\zeta] = (l + 2m + n)/4$.

Theorem 2. *(Liu [20]) The expected value operator satisfies the linearity property $E[x\zeta + y\xi] = xE[\zeta] + yE[\xi]$, where ξ and ζ are any two independent uncertain variables and $x, y \in \mathcal{R}$.*

Definition 6. *(Liu [20]) The γ-pessimistic and γ-optimistic values of ζ are defined by*

$$\zeta_{inf}(\gamma) = inf\{t|\mathcal{M}\{\zeta \leq t\} \geq \gamma\} = \Psi^{-1}(\gamma),\ \gamma \in (0,1].$$

$$\zeta_{sup}(\gamma) = sup\{t|\mathcal{M}\{\zeta \geq t\} \geq \gamma\} = \Psi^{-1}(1-\gamma),\ \gamma \in (0,1].$$

For zigzag uncertain variable $\mathcal{Z}(l,m,n)$, we have:

$$\zeta_{sup}(\gamma) = \begin{cases} 2\gamma m + (1-2\gamma)n, & \text{if } \gamma < 0.5, \\ (2\gamma - 1)l + (2-2\gamma)m, & \text{if } \gamma \geq 0.5, \end{cases} \tag{1}$$

$$\zeta_{inf}(\gamma) = \begin{cases} (1-2\gamma)l + 2\gamma m, & \text{if } \gamma < 0.5, \\ (2-2\gamma)m + (2\gamma - 1)n, & \text{if } \gamma \geq 0.5. \end{cases} \tag{2}$$

Theorem 3. *(Liu [20]) Let ζ be an uncertain variable and $\gamma \in (0,1]$. Then we have*

(a) *$\zeta_{inf}(\gamma)$ is left-continuous and increasing function of γ.*
(b) *$\zeta_{sup}(\gamma)$ is left-continuous and decreasing function of γ.*

The fundamental problem seen in handling the uncertain variables is how to rank the uncertain numbers as there is no specific order in the uncertain environment. For this reason, four criteria were introduced by Liu [44] to rank the uncerain numbers. These ranking criteria are: Expected Value Criterion (EVC), Optimistic Value Criterion (OVC), Pessimistic Value Criterion (PVC) and Chance-Criterion (CC). Considering two uncertain variables ζ and ξ, he stated these ranking criteria as:

EVC states that $\zeta < \xi$ iff $E[\zeta] < E[\xi]$.
OVC states that $\zeta < \xi$ iff $\zeta_{sup}(\gamma) < \xi_{sup}(\gamma)$, for some $\gamma \in (0,1]$.
PVC states that $\zeta < \xi$ iff $\zeta_{inf}(\gamma) < \xi_{inf}(\gamma)$, for some $\gamma \in (0,1]$.
CC states that $\zeta < \xi$ iff $\mathcal{M}\{\zeta \geq \bar{t}\} < \mathcal{M}\{\xi \geq \bar{t}\}$ for some predefined level $\bar{t}$.

3. Problem description

This section describes the MCMOTP with an assumption of m sources and n destinations. MCMOTP concerns with developing an ideal transportation plan with an objective of obtaining the minimum of the objective vector Z^t consisting of different objectives like transportation cost, damage cost, transportation time etc.

The MCMOTP model can be mathematically formulated as given below:

Model-1:

$$\min\ Z^t = \sum_{\forall i}\sum_{\forall j}(c_{ij}^t x_{ij}), \forall t;$$

Subject to the given constraints:

$$\sum_{\forall j} x_{ij} \le a_i,\ \forall i, \tag{3.1}$$

$$\sum_{\forall i} x_{ij} \ge b_j,\ \forall j, \tag{3.2}$$

$$x_{ij} \ge 0; \tag{3.3}$$

Here, we have have used notations $\forall\ t$ for $t = 1, 2, \cdots S$, $\forall\ i$ for $i = 1, 2, \cdots m$ and $\forall\ j$ for $j = 1, 2, \cdots n$ throughtout this paper, with S, m and n representing the total number of objectives, sources and destinations respectively. a_i represents the supplying capacity of the source i and b_j represents the demand requirements at destination j. The notation c_{ij}^t is used for representing different objective parameters like shipping cost, damage cost for a unit item to destination j from source i corresponding to objective t whereas x_{ij} represents the number of items transported from source i to destination j.

The above Model-1 assumes all the variables a_i, b_j, c_{ij}^t as constants. But in the practical situations, we are not able to define these variables accurately due to lack of information as the transportation plan is supposed to be made in advance. If the previously used information regarding the plan is available, the variables can be treated as the random variables but if we are not provided with the previous information then treating these variables as the random variables will not lead us to the appropriate results. Thus, in such cases, when we have lack of information about the historical data, we take into consideration the concepts of uncertainty theory given by Liu [20] in the objective functions.

Furthermore, the decision-makers face more complexities in making a decision when there are multiple choices/alternatives in the TP for parameters such as cost, demand and supply. These multiple alternatives for the parameters may exist due to several routes for transporting the goods or effect of climatic conditions during transportation. Therefore, in this problem, we have considered the supply and demand a_i, b_j as multi-choice parameters $\left(a_i^{(1)}, a_i^{(2)}, \cdots a_i^{(k_i)}\right)$, $\left(b_j^{(1)}, b_j^{(2)}, \cdots b_j^{(k_j)}\right)$ and objectives c_{ij}^t as uncertain objectives denoted by ζ_{ij}^t. So, the MCMOTP becomes Uncertain MCMOTP, denoted by UMCMOTP.

4. Uncertain model of MCMOTP with multi-choices in constraints

Replacing the uncertain variables ζ_{ij}^t and multi-choices $\left(a_i^{(1)}, a_i^{(2)}, \cdots a_i^{(k_i)}\right)$, $\left(b_j^{(1)}, b_j^{(2)}, \cdots b_j^{(k_j)}\right)$ in the objective functions and constraints of Model-1, to get the following UMCMOTP Model-2:

Model-2:

$$\min\ Z^t(x;\zeta) = \sum_{\forall i}\sum_{\forall j}\zeta_{ij}^t x_{ij}, \forall t; \tag{4.1}$$

Subject to the given constraints:

$$\sum_{\forall j} x_{ij} \le \left(a_i^{(1)}, a_i^{(2)}, \cdots a_i^{(k_i)}\right),\ \forall i, \tag{4.2}$$

$$\sum_{\forall i} x_{ij} \ge \left(b_j^{(1)}, b_j^{(2)}, \cdots b_j^{(k_j)}\right),\ \forall j, \tag{4.3}$$

$$x_{ij} \ge 0; \tag{4.4}$$

which is called the uncertain programming model.

Since this uncertain mathematical model is difficult to handle due to the presence of uncertain objectives and multi-choice constraints, we need to convert both of them (uncertain objectives and multi-choice constraints) into their deterministic forms as discussed below.

4.1. Conversion procedure for multi-choice constraints

Here, we have considered the situation when the supply or demand values are not defined by an exact number. In this framework, the use of multi-choices should therefore be used to define the value of supply and demand. As there is no existing method in the literature to handle multi-choices, Biswal and Acharya [30] provided

a transformation technique for obtaining the deterministic form of the multi-choice constraints in the MOTP Model-2 because there must be a single choice for all the parameters to solve the model. Assume that the supply capacity a_i at the source i has multiple choices represented in the form $\left(a_i^{(1)}, a_i^{(2)}, \cdots a_i^{(k_i)}\right), \forall i$, where k_i is the number of multiple choices available for the parameter a_i.
Let us consider the multi-choice supply constraint (4.5) with k_i choices of supply capacity at the i^{th} origin is written as follows:

$$\sum_{j=1}^{n} x_{ij} \leq \left(a_i^{(1)}, a_i^{(2)}, \cdots a_i^{(k_i)}\right), \forall i. \tag{4.5}$$

The transformation technique presented by Biswal and Acharya [30] considers two classes of k_i for obtaining a single choice from multi-choice parameters. The first class deals with the case when $k_i = 2^{m_1}$ and the second class deals with the case when $k_i \neq 2^{m_1}$ with $m_1 = 1, 2, 3, 4$. In his paper, he has considered all the fifteen cases when $2 \leq k_i \leq 16$. Here, we have discussed only three cases of k_i taken as $k_i = 2, 3, 4$ which are shown as below:
when $k_i = 2$, the multi-choice supply constraint is represented as:

$$\sum_{j=1}^{n} x_{ij} \leq \left(a_i^{(1)}, a_i^{(2)}\right), \forall i. \tag{4.6}$$

There are two multiple choices $a_i^{(1)}, a_i^{(2)}$ for the parameter a_i from which a single choice is to be chosen. Since there are two elements in the set $\left(a_i^{(1)}, a_i^{(2)}\right)$, we need only a single binary variable $m_i^{(1)}$ to handle these two choices. Using the binary variable $m_i^{(1)}$, the constraint (4.6) can be converted to the following constraint (4.7) formulated as given below:

$$\sum_{j=1}^{n} x_{ij} \leq m_i^{(1)} a_i^{(1)} + (1 - m_i^{(1)}) a_i^{(2)}, \forall i, \tag{4.7}$$

$$m_i^{(1)} \in \{0, 1\}, \forall i.$$

With $k_i = 3$, the three multi-choices in the supply constraint are represented as:

$$\sum_{j=1}^{n} x_{ij} \leq \left(a_i^{(1)}, a_i^{(2)}, a_i^{(3)}\right), \forall i. \tag{4.8}$$

There are three known parameters $a_i^{(1)}, a_i^{(2)}, a_i^{(3)}$ in the right hand side of the equation (4.8) from which a single choice is to be selected. To deal with the multi-choice constraint (4.8), we need only two variables $m_i^{(1)}$ & $m_i^{(2)}$ because $2^1 < 3 < 2^2$. Using these two binary variables $m_i^{(1)}$ & $m_i^{(2)}$, we can transform the multi-choice constraint (4.8) to constraint (4.9) along with an additional constraint (4.10) for restricting the two binary variables $m_i^{(1)}$ & $m_i^{(2)}$.

$$\begin{aligned}\sum_{j=1}^{n} x_{ij} \leq & \left(1 - m_i^{(1)}\right)\left(1 - m_i^{(2)}\right) a_i^{(1)} + \left(1 - m_i^{(1)}\right) m_i^{(2)} a_i^{(2)} \\ & + m_i^{(1)} \left(1 - m_i^{(2)}\right) a_i^{(3)}, \forall i,\end{aligned} \tag{4.9}$$

$$m_i^{(1)} + m_i^{(2)} \leq 1, \tag{4.10}$$

$$m_i^{(p)} \in \{0, 1\}, p = 1, 2,$$
$$x_{ij} \geq 0, \forall i, \forall j.$$

With $k_i = 4$, the four multi-choices in the supply constraint can be represented as:

$$\sum_{j=1}^{n} x_{ij} \leq \left(a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right), \forall i. \tag{4.11}$$

To handle the four known parameters $a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}$ in the right hand side of constraint (4.11), we require only two binary variables $m_i^{(1)}$ and $m_i^{(2)}$ because there are 2^2 choices in the constraint (4.11) out of which one of them is to be selected. Using these two binary variables $m_i^{(1)}$ and $m_i^{(2)}$, the constraint (4.11) can be converted into the following constraint (4.12):

$$\begin{aligned}\sum_{j=1}^{n} x_{ij} \leq & m_i^{(1)} m_i^{(2)} a_i^{(1)} + \left(1 - m_i^{(1)}\right) m_i^{(2)} a_i^{(2)} \\ & + m_i^{(1)} \left(1 - m_i^{(2)}\right) a_i^{(3)} \\ & + \left(1 - m_i^{(1)}\right)\left(1 - m_i^{(2)}\right) a_i^{(4)}, \forall i \\ & m_i^{(p)} \in \{0, 1\}, p = 1, 2, \\ & x_{ij} \geq 0, \forall i, \forall j.\end{aligned} \tag{4.12}$$

Similar to multiple choices for supply capacity parameters, the demand parameters can also be multi-choices involved due to factors like seasonality, taxation, product availability and pricing. The procedure of transforming the multi-choice demand constraint $\sum_{i=1}^{m} x_{ij} \geq \left(b_j^{(1)}, b_j^{(2)}, \cdots b_j^{(k_j)}\right), j = 1, 2 \cdots n$ is same as stated for multi-choice supply points.

4.2. Conversion procedure for uncertain objectives

The objectives in Model-2 consists of S different objectives like uncertain transportation cost and damage cost etc., represented by ζ_{ij}^t which are uncertain variables. As stated earlier, the basic problem in uncertain transportation problems is how to rank the uncertain variables. Thus, Liu [44] introduced four ranking criteria as: EVC, OVC, PVC and CC discussed in Section 2.

These four ranking criteria can be used to convert the uncertain objectives into their deterministic forms. In this paper, we have only utilized the expected value criterion and optimistic value criterion for converting the uncertain objectives into crisp objectives but the other two remaining criterions can also be utilized for the conversion of uncertain objectives.

a) Expected value criterion: The main idea in this criterion turns out to utilize the expected values of the uncertain variables in the objective functions. So, the expected form of the uncertain objective (4.1) in Model-2 can be written as:

$$\min\ E[Z_t] = E\left[\sum_{\forall i}\sum_{\forall j}\zeta_{ij}^t x_{ij}\right], \forall t; \quad (4.13)$$

Theorem (1) is now used to deduce the crisp formulation of the objective as shown in equation (4.14). This crisp objective can be considered for formulating the mathematical model along with the crisp form of the multi-choice constraints (as discussed in Section 4.1). The mathematical Model-3, also known as the expected value model, is a crisp model of the uncertain Model-2.

Model-3:

$$\min\ Z_{tE} = \sum_{\forall i}\sum_{\forall j}\left(x_{ij}\int_0^1 \Psi_{\zeta_{ij}^t}^{-1}(\eta_t)d\eta_t\right), \forall t; \quad (4.14)$$

Subject to the constraints (4.2) to (4.4).

b) Optimistic value criterion: To deal with uncertain objectives, optimistic value criterion can also be utilized to convert the uncertain variables in crisp form.

$$\min\ Z_{sup}^t(\eta_t) = \left[\sum_{\forall i}\sum_{\forall j}\zeta_{ij}^t x_{ij}\right]_{sup}(\eta_t), \forall t; \quad (4.15)$$

The optimistic value based-objective function (4.15) can further be simplified using the equation (1) and can be equivalently written as shown in equation (4.16). The mathematical Model-4 formed with the crisp objective function (4.16) along with the crisp multi-choice constraints is labelled as optimistic value model. In objective functions (4.15), η_t are the confidence levels assumed with some fixed values.

Model-4:

$$\min\ Z_{tS} = \sum_{\forall i}\sum_{\forall j} x_{ij}\Psi_{\zeta_{ij}^t}^{-1}(1-\eta_t), \forall t; \quad (4.16)$$

Subject to the constraints (4.2) to (4.4).

5. Solution approaches

In this section, two main classical approaches are discussed for obtaining the compromise solution of the multi-objective optimization problems which are Minimizing Distance Method (MDM) and fuzzy programming technique. These two methods are utilized to obtain the compromise solution for the crisp formulations of EV and OV Models.

5.1. Minimizing distance method

This method transforms a multi-objective problem into single objective by minimizing the sum of deviation of the ideal vector from the corresponding objective functions. In this method, Euclidean distance is used to convert the crisp multi-objective models i.e. EV Model and OV Model into their equivalent compromise model as given below:

$$\min \sqrt{\sum_{t=1}^{S}(Z_t - Z_t^o)^2}$$

subject to the constraints (4.2) to (4.4).

Here, Z_t is a generalized representation for the objective functions in both the models and Z_t^o denote the ideal objective value of the t^{th} objective function in EV and OV Models without considering other objective functions.

5.2. Fuzzy programming technique

The fuzzy programming approach introduced by Zimmerman [5] is applicable to multi-objective problems only and their solutions can be obtained using the sequential steps as defined below:

Step 1. Consider each objective function of the deterministic Models 3 and 4 as a single objective problem by ignoring all the other objectives and proceed to the next step.

Step 2. Obtain the minimum (L_t) and maximum (U_t) values for $t = 1, 2, \cdots S$ objective functions.

Step 3. Formulate the linear or exponential membership function $\mu_t(Z_t)$ for $t = 1, 2, \cdots S$ objective functions:

A Linear Membership Function (LMF) can be defined by:

$$\mu_t(Z_t) = \begin{cases} 1, & \text{if } Z_t \leq L_t, \\ \frac{U_t - Z_t}{U_t - L_t}, & \text{if } L_t < Z_t < U_t, \\ 0, & \text{if } Z_t \geq U_t, \quad \forall t. \end{cases} \tag{5.1}$$

An Exponential Membership Function (EMF) is defined by:

$$\mu_t(Z_t) = \begin{cases} 1, & \text{if } Z_t \leq L_t, \\ \dfrac{e^{-s_t \psi_t(x)} - e^{-s_t}}{1 - e^{-s_t}}, & \text{if } L_t < Z_t < U_t, \\ 0, & \text{if } Z_t \geq U_t, \quad \forall t. \end{cases} \tag{5.2}$$

where, $\psi_t(x) = \frac{Z_t - L_t}{U_t - L_t}$ and s_t is a non-zero shape parameter given by the decision-maker.

Step 4. Using the max-min operator, formulate the single-objective model as:

$$\text{Maximize } \lambda$$

Subject to the given constraints:

$$\mu_t(Z_t) \geq \lambda, \text{where } \lambda = \min \mu_t(Z_t) \geq 0. \ t = 1, 2,$$
$$\text{and the constraints (4.2) to (4.4) },$$

Step 5. This single-objective model is further solved in LINGO 18.0 optimization tool to achieve the compromise solution of the MCMOTP problem.

6. Numerical illustration of uncertain MCMOTP

Let us consider a TP in which we have three origins ($m = 3$) and three destinations ($n = 3$). In this problem, all the parameters involved with objectives like transportation cost/damage cost are all considered as independent zigzag uncertain variables. The other parameters like supplier capacities and destination demands are the parameters with multi-choices. The problem aims at finding the total number of products to be shipped from sources to destinations such that the transportation cost and damage cost of items during transportation is minimized. The uncertain data for the objectives is given in Table 1 and the multi-choices for the capacity of suppliers and demands required at destinations are listed below:

$\tilde{a}_1 \in \{8, 10, 12\}$, $\tilde{a}_2 \in \{9, 10, 11, 13\}$,
$\tilde{a}_3 \in \{12, 14\}$, $\tilde{b}_1 \in \{7, 8\}$,
$\tilde{b}_2 \in \{6, 7, 8\}$, $\tilde{b}_3 \in \{9, 10, 11\}$

Table 1. The shipping/damage costs for two objectives from i sources to j destinations.

ζ^1_{ij}	1	2	3	ζ^2_{ij}	1	2	3
1	(2,3,4)	(5,6,7)	(4,6,8)	1	(6,8,9)	(4,6,7)	(6,8,10)
2	(3,5,7)	(1,3,5)	(2,3,4)	2	(3,5,7)	(2,3,4)	(7,8,9)
3	(4,6,8)	(7,8,9)	(6,8,10)	3	(8,9,10)	(5,6,7)	(6,7,8)

The uncertain problem defined above consists of uncertain objectives and multi-choice constraints. The uncertain model formed with this uncertain data can be converted into its crisp models: EV and OV Models using the procedure defined in Section 4.1.These two models are solved here and their results are obtained with the two solution methodologies mentioned in Section 5.

Solution:

6.1. Expected value model

The objective functions of the Model-5 are obtained by applying the expected value operator on the zigzag uncertain variables given in the Table 1. The multi-choice constraints in Model-5 are further converted to single choice constraints using the procedure given by Biswal and Acharya [30] as described in Section 4.1.

Model-5:

$$\begin{aligned} \min Z_{1E} =& 3x_{11} + 6x_{12} + 6x_{13} + 5x_{21} + 3x_{22} + 3x_{23} + 6x_{31} \\ &+ 8x_{32} + 8x_{33}; \\ \min Z_{2E} =& 7.75x_{11} + 5.75x_{12} + 8x_{13} + 5x_{21} + 3x_{22} + 8x_{23} \\ &+ 9x_{31} + 6x_{32} + 7x_{33}; \end{aligned}$$

Subject to the given constraints:

$$\begin{aligned} &x_{11} + x_{12} + x_{13} \leq \{8, 10, 12\}; \\ &x_{21} + x_{22} + x_{23} \leq \{9, 10, 11, 13\}; \\ &x_{31} + x_{32} + x_{33} \leq \{12, 14\}; \\ &x_{11} + x_{21} + x_{31} \geq \{7, 8\}; \\ &x_{12} + x_{22} + x_{32} \geq \{6, 7, 8\}; \\ &x_{13} + x_{23} + x_{33} \geq \{9, 10, 11\}; \\ &x_{ij} \geq 0, \forall i, \forall j; \end{aligned}$$

The results obtained for the EV Model using the discussed two solution methodologies are presented here.

a) Minimizing distance method:

The results obtained using MDM are displayed below with ideal values taken as $Z^o_{1E} = 72$ and $Z^o_{2E} = 116$:

$Z^*_{1E} = 83.92890, Z^*_{2E} = 137.6891, x_{11} = 4.6142, x_{13} = 4.3858,$
$x_{21} = 2.3858, x_{22} = 6, x_{23} = 4.6142$

b) Fuzzy programming technique:

i) Using linear membership function

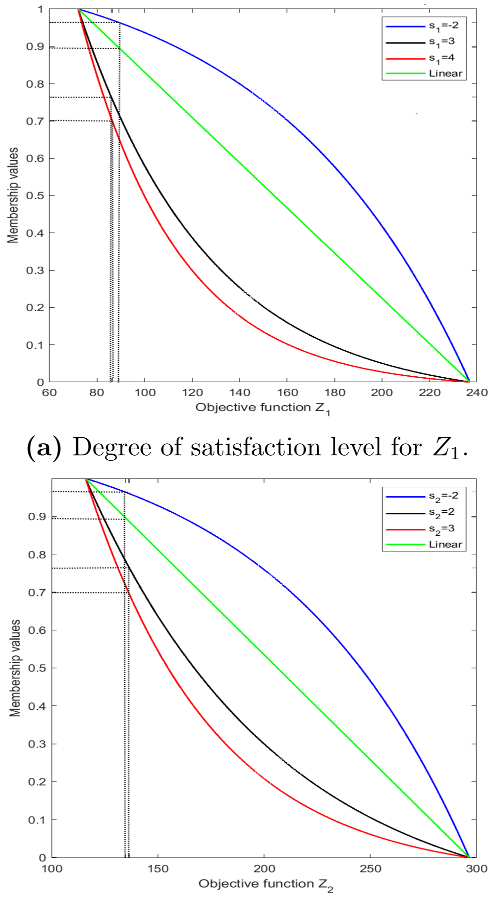
To apply the FPT with linear membership function, the U_t and L_t are obtained as: $L_1 = 72,\ L_2 = 116,\ U_1 = 237,\ U_2 = 296.5$ which are substituted in linear membership function (5.1) to get a single objective fuzzy model as discussed in step 5 of Section 5.2. The solution obtained for the EV Model-5 by applying the fuzzy technique procedure is: $\lambda = 0.8958525, x_{11} = 3.563134, x_{13} = 5.436866, x_{21} = 3.436866, x_{22} = 6, x_{23} = 3.563134$ and the compromise solution for the objective functions are $Z^*_{1E} = 89.18433$ and $Z^*_{2E} = 134.7986$.

Here, λ represents the minimum value amongst both the membership functions. i.e. $\lambda = \min\ (\lambda_1,\ \lambda_2)$, where $\lambda_1 = \mu_1(Z_{1E})$ and $\lambda_2 = \mu_2(Z_{2E})$.

ii) Using exponential membership function

For applying FPT with exponential membership function, the U_t and L_t values are $L_1 = 72,\ L_2 = 116,\ U_1 = 237,\ U_2 = 296.5$ used to construct an exponential membership function given in (5.2) for formulating a single objective fuzzy model discussed in step (5) of Section 5.2 for the EV Model-5. Solving the single objective fuzzy model of EV Model-5 for three different cases of shape parameters (s_1, s_2) taken as $(-2, -2)$, $(3, 2)$ and $(4, 3)$ in the exponential membership function, we get the solutions as shown in Table 2.

The graphical representation of the objective functions with respect to the linear and exponential (with three cases of shape parameters) membership functions in EV Model-5 is shown in Figure 1. In Figure 1, it is clearly seen that the compromise solution of the objective functions is achieved at their corresponding degree of satisfaction level. For example, in the case of linear membership function, the compromise solution $Z^*_{1E} = 89.18433$ and $Z^*_{2E} = 134.7986$ are obtained at membership values $\lambda_1 = 0.8958525$ and $\lambda_2 = 0.8958525$ which represents the individual degree of satisfaction for objectives Z_{1E} and Z_{2E}. The fuzzy programming solution approach gives the minimum value λ of these individual membership function values $\lambda_1,\ \lambda_2$, stating that each of the objective function posseses at least λ degree of satisfaction level. As the membership value increases and approaches to 1 for the objective functions, the objective values are improved simultaneously and approach to the best value (optimal value) of the individual objective functions. Also, for each objective function, the shape parameter values can be chosen at random until the model produces a feasible solution. Changing the shape parameter values will result in different compromise solutions.



(a) Degree of satisfaction level for Z_1.

(b) Degree of satisfaction level for Z_2.

Figure 1. Graphical representation of solutions for EV Model-5 with FPT.

6.2. Optimistic value model

To formulate OV Model, we need the predetermined confidence levels $\eta_t \in (0, 1]$. Let us assume that all the confidence levels are equal to 0.9.

Model-6:

$$\begin{aligned}\min Z_{1S} =& 2.2x_{11} + 5.2x_{12} + 4.4x_{13} + 3.4x_{21} + 1.4x_{22} + 2.2x_{23}\\ &+ 4.4x_{31} + 7.2x_{32} + 6.4x_{33};\\ \min Z_{2S} =& 6.4x_{11} + 4.4x_{12} + 6.4x_{13} + 3.4x_{21} + 2.2x_{22} + 7.2x_{23}\\ &+ 8.2x_{31} + 5.2x_{32} + 6.2x_{33};\end{aligned}$$

Table 2. Solution table for (s_1, s_2) in the exponential membership function for EV Model-5.

(s_1, s_2)	Z^*_{1E}	Z^*_{2E}	$\lambda = \lambda_1 = \lambda_2$	Solution
(-2,-2)	89.18433	134.7986	0.963754	$x_{11} = 3.563134, x_{13} = 5.436866, x_{21} = 3.436866, x_{22} = 6, x_{23} = 3.563134$
(3,2)	85.95132	136.5768	0.764216	$x_{11} = 4.209737, x_{13} = 4.790263, x_{21} = 2.790263, x_{22} = 6, x_{23} = 4.209737$
(4,3)	86.46528	136.2941	0.698695	$x_{11} = 4.106944, x_{13} = 4.893056, x_{21} = 2.893056, x_{22} = 6, x_{23} = 4.106944$

Subject to the given constraints of Model-5. The solution of this multi-objective OV Model-6 can be achieved with the two solution methodologies mentioned in section 5. The results of both the methods are shown below.

a) Minimizing distance method:
The solution obtained for Model-6 using MDM is:

$$Z^*_{1S} = 62.1126, Z^*_{2S} = 105.4271, x_{11} = 2.8492, x_{13} = 6.1508,$$
$$x_{21} = 4.1508, x_{22} = 6, x_{23} = 2.8492$$

with ideal values of the objective functions taken as $Z^o_{1S} = 48$ and $Z^o_{2S} = 92.8$.

b) Fuzzy programming technique:

i) Using linear membership function

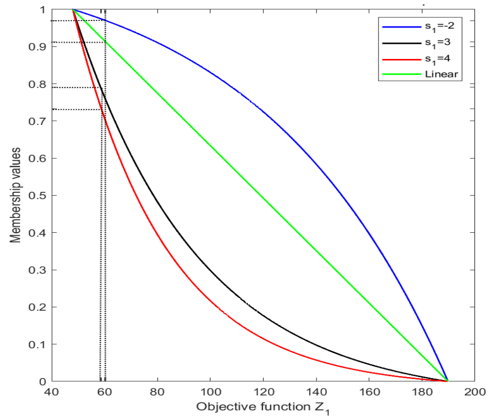
The sequential steps of the FPT can be applied to obtain the U_t and L_t values as: $L_1 = 48$, $L_2 = 92.8$, $U_1 = 189.80$, $U_2 = 260.40$. The solution obtained for the OV Model-6 using linear membership function is given as: $\lambda = 0.9129054, x_{11} = 3.367644, x_{13} = 5.632356, x_{21} = 3.632356, x_{22} = 6, x_{23} = 3.367644$ and the compromise solution for the objective functions are $Z^*_{1S} = 60.35001$ and $Z^*_{2S} = 107.3970$. Here, λ represents the minimum value amongst all these membership functions. i.e. $\lambda = \min(\lambda_1, \lambda_2)$, where $\lambda_1 = \mu_1(Z_{1S})$ and $\lambda_2 = \mu_2(Z_{2S})$.

ii) Using exponential membership function

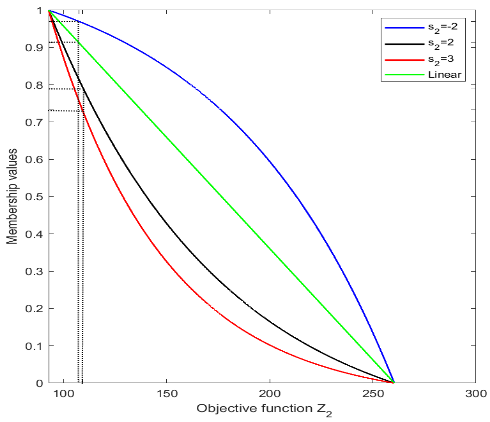
The solutions obtained for three different cases of shape parameters $(s_1 = -2, s_2 = -2)$,$(s_1 = 3, s_2 = 2)$ and $(s_1 = 4, s_2 = 3)$ are displayed in Table 3.

The graphical representation of the linear and exponential membership function versus the objective functions in Model-6 is shown in Figure 2.

In Figure 2, it is observed that the compromise solution of both the objective functions (with linear and exponential membership function in FPT) is achieved at their corresponding individual degree of satisfaction. Say, for the linear membership function, the compromise values of the objective functions $Z^*_{1S} = 60.35001$ and $Z^*_{2S} = 107.3970$ are achieved at degree of satisfactions $\lambda_1 = 0.9129054$ and $\lambda_2 = 0.9129054$. Similarly, the compromise solution of both objective functions with three cases of shape parameters in exponential membership functions is achieved at their respective individual degrees of satisfaction, as can be seen in Figure 2.



(a) Degree of satisfaction level for Z_1.



(b) Degree of satisfaction level for Z_2.

Figure 2. Graphical representation of solutions for OV Model-6 with FPT.

6.2.1. Sensitivity analysis of the objective functions in OV model

Here, the sensitivity of the objective functions is investigated in the OV Model with respect to the confidence level η_t. The complementary test is performed by variating the values of the confidence level η_t in the range $[0.1, 0.9]$ with a step

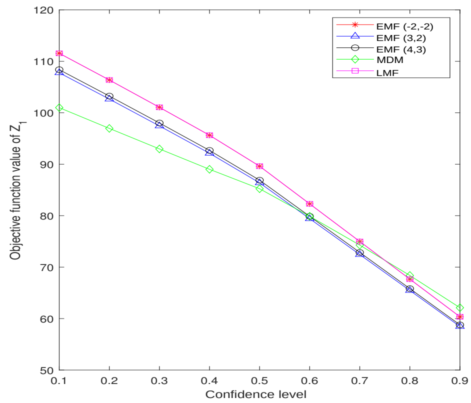
Table 3. Solution table for (s_1, s_2) in the exponential membership function for OV Model-6

(s_1, s_2)	Z^*_{1S}	Z^*_{2S}	$\lambda = \lambda_1 = \lambda_2$	Solution
(-2,-2)	60.35001	107.3970	0.970218	$x_{11} = 3.367644, x_{13} = 5.632356, x_{21} = 3.632356, x_{22} = 6, x_{23} = 3.367644$
(3,2)	58.46478	109.5041	0.790991	$x_{11} = 3.922123, x_{13} = 5.077877, x_{21} = 3.077877, x_{22} = 6, x_{23} = 3.922123$
(4,3)	58.77858	109.1534	0.732933	$x_{11} = 3.829830, x_{13} = 5.170170, x_{21} = 3.170170, x_{22} = 6, x_{23} = 3.829830$

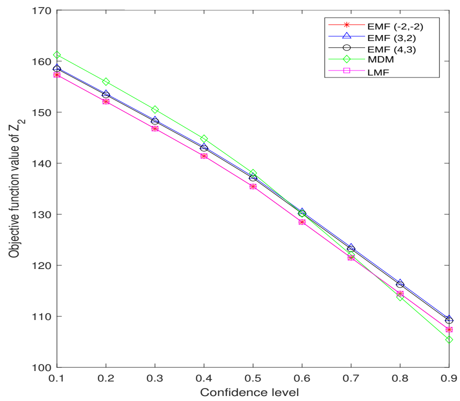
size of 0.1 with respect to the crisp multi-choice constraints of Model-6. The results of the sensitivity analysis are shown for both the two solution methodologies. The objective values obtained during the sensitivity analysis of OV Model-6 are shown in Table 4. "CL" represents the variation of confidence level in $\eta_t : t = 1, 2$, for both the objective functions.

Table 4. Objective values obtained during the sensitivity analysis of the OV Model-6 with FPT.

CL	Z_{tS}	MDM	Fuzzy programming technique			
			EMF			LMF
			(-2,-2)	(3,2)	(4,3)	
0.1	Z_{1S}	100.9969	111.5599	107.7755	108.3744	111.5599
	Z_{2S}	161.2291	157.2987	158.7068	158.4839	157.2987
0.2	Z_{1S}	96.95949	106.3698	102.6568	103.2445	106.3698
	Z_{2S}	155.9729	152.0711	153.6106	153.3669	152.0711
0.3	Z_{1S}	92.97171	101.0594	97.4410	98.01394	101.0594
	Z_{2S}	150.5054	146.7726	148.4426	148.1782	146.7726
0.4	Z_{1S}	89.01075	95.63106	92.12848	92.68335	95.63106
	Z_{2S}	144.7999	141.4003	143.1989	142.9140	141.4003
0.5	Z_{1S}	85.23529	89.61566	86.37785	86.89344	89.61566
	Z_{2S}	138.0588	135.4306	137.3733	137.0639	135.4306
0.6	Z_{1S}	79.87618	82.29727	79.41449	79.87810	82.29727
	Z_{2S}	130.1470	128.4628	130.4682	130.1457	128.4628
0.7	Z_{1S}	74.2800	74.98182	72.44310	72.85571	74.98182
	Z_{2S}	122.0400	121.4719	123.5270	123.1930	121.4719
0.8	Z_{1S}	68.37898	67.66704	65.46099	65.82364	67.66704
	Z_{2S}	113.7778	114.4523	116.5422	116.1987	114.4523
0.9	Z_{1S}	62.11262	60.35001	58.46478	58.77854	60.35001
	Z_{2S}	105.4271	107.3970	109.5041	109.1534	107.3970



(a) Sensitivity analysis of Z_1 w.r.t η_t in the Model-6.



(b) Sensitivity analysis of Z_2 w.r.t η_t in the Model-6.

Figure 3. The sensitivity analysis of the objectives in OV Model-6 w.r.t η_t using FPT and MDM.

The graphical interpretation of the objective values w.r.t the confidence level η_t are shown in Figure 3. Figure 3 indicates that the objective function values are decreasing with respect to the tested confidence levels η_t for both the solution methodologies.

7. Comparison of the results

This section presents the results obtained for the Uncertain MCMOTP using the EV and OV Models. Table 5 compares the results obtained for EV and OV models with minimizing distance method and fuzzy programming technique methodologies.

Table 5. Comparison of the results obtained with two methodologies.

Model	Z_t	MDM	Fuzzy programming technique			
			EMF			LMF
			(-2,-2)	(3,2)	(4,3)	
EV Model	Z_{1E}^*	83.9290	89.1843	85.9513	86.4653	89.1843
	Z_{2E}^*	137.6891	134.7986	136.5768	136.2941	134.7986
OV Model	Z_{1S}^*	62.1126	60.3500	58.4648	58.7786	60.3500
	Z_{2S}^*	105.4271	107.3970	109.5041	109.1534	107.3970

From the results obtained with the given solution methodologies, it can be observed that neither of the method is dominating the results of the other method because if one objective approaches towards its best value then the other objective value starts worsening. Also, the EV Model gives the solution in terms of expected values of the objective functions, OV Model gives the solution in terms of optimistic values of the objective functions. The results of the OV Model obtained here are only for a single case of confidence level $\eta_t = 0.9$ in the objective function, but varying η_t in the range $(0, 1]$ can provide numerous set of solutions.

8. Conclusion

This paper developed the Uncertain Multi-choice MOTP with objectives as zigzag uncertain variables and supply and demand parameters as multi-choice parameters. The uncertain MC-MOTP model has been solved using the two crisp models: EV and OV Models. Further, the crisp models were solved using minimizing distance method and fuzzy technique (with linear and exponential membership functions). The EV Model will always lead to a single or multiple solution based on the solution methodology utilized whereas OV Model will always provide numerous solutions to the decision maker because of the confidence level η_t involved in the OV Model. In comparison to the EV Model, which does not incorporate confidence level, the OV Model may provide the decision maker with a number of alternative solutions by altering the values of the confidence level η_t between 0 and 1.

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